II. *Æquationum Cubicarum & Biquadraticarum, tum Analytica, tum Geometrica & Mechanica, Reſolutio Univerſalis, a* J. Colſon.

§. 1. Æquationis Cubicæ Univerſalis $\left\{ \begin{array}{r} x^3 = 3px^2 + 3qx + 2r, \\ -3p^2 + p^3 \\ -3pq \end{array} \right.$

Radices Tres ſunt,

$$x = p + \sqrt[3]{r + \sqrt{r^2 - q^3}} + \sqrt[3]{r - \sqrt{r^2 - q^3}}$$

$$x = p - \frac{1 - \sqrt{-3}}{2} \times \sqrt[3]{r + \sqrt{r^2 - q^3}} - \frac{1 + \sqrt{-3}}{2} \times \sqrt[3]{r - \sqrt{r^2 - q^3}}$$

$$x = p - \frac{1 + \sqrt{-3}}{2} \times \sqrt[3]{r + \sqrt{r^2 - q^3}} - \frac{1 - \sqrt{-3}}{2} \times \sqrt[3]{r - \sqrt{r^2 - q^3}}$$

Vel ut Calculus Arithmeticus facilior ac paratior evadat, ſi poſueris Binomii irrationalis $r + \sqrt{r^2 - q^3}$ Radicem Cubicam eſſe $m + \sqrt{n}$, erunt ejuſdem Æquationis Radices tres $x = p + 2m$, & $x = p - m \pm \sqrt{-3n}$.

Igitur data Æquatione quavis Cubica, inter ejus hujuſque Æquationis Univerſalis terminos ſingulos inſtituenda eſt comparatio, quo pacto facillime invenientur ipſæ p, q, r; & hiſce cognitis, innoteſcent Æquationis datæ Radices omnes. Hujus vero Solutionis Exempla ſint ſequentia in Numeris.

1. Æquationis Cubicæ $x^3 = 2x^2 + 3x + 4$ ſit Radix x indaganda. Erit primò juxta præſcriptum $3p = 2$,

ſive $p=\frac{2}{3}$. Secundò $3q-(3p^2)\frac{4}{3}=3$, ſive $q=\frac{13}{9}$. Tertiò $2r(+\sqrt{p^2-3q}\times p)-\frac{70}{27}=4$, ſive $r=\frac{89}{27}$, & $r^2-q^3=\frac{212}{27}$. Et propterea $x=\frac{2}{3}+\sqrt[3]{\frac{89}{27}+\sqrt{\frac{212}{27}}}+\sqrt[3]{\frac{89}{27}-\sqrt{\frac{212}{27}}}$. Reliquæ duæ Radices ſunt impoſſibiles.

2. In Æquatione $x^3=12x^2-41x+42$, erit primò $3p=12$, ſive $p=4$. Secundò $3q-(3p^2)48=-41$, ſive $q=\frac{7}{3}$. Tertiò $2r+(\sqrt{p^2-3q}\times p)36=42$, ſive $r=3$; Et inde $r^2-q^3=-\frac{100}{27}$. At Binomii ſurdi $3+\sqrt{-\frac{100}{27}}(=r+\sqrt{r^2-q^3})$ Radix Cubica, per Methodos ex Arithmetica petendas extracta, eſt $-1+\sqrt{-\frac{4}{3}}$ $(=m+\sqrt{n},)$ & proinde Radix $x=(p+2m=4-2=)2$, vel etiam $x=(p-m\pm\sqrt{-3n}=4+1\pm(\sqrt{4})2=)7$ vel 3. Vel rurſus, ejuſdem Binomii $3+\sqrt{-\frac{100}{27}}$. Radix alia Cubica (tres enim agnoſcit) eſt $\frac{3}{2}+\sqrt{-\frac{1}{12}}$ $(=m+\sqrt{n},)$ & proinde Radix $x=(p+2m=4+3=)7$, & etiam $x=(p-m\pm\sqrt{-3n}=4-\frac{3}{2}\pm(\sqrt{\frac{1}{4}})\frac{1}{2}=3$ vel 2. Vel denuo, ejuſdem Binomii $3+\sqrt{-\frac{100}{27}}$ Radix Cubica tertia eſt $-\frac{1}{2}-\sqrt{-\frac{25}{12}}$, $(=m+\sqrt{n},)$ & proinde Radix

$x=$

$x = (p + 2m = 4 - 1 =) 3$, atque etiam $x = (p - m \pm \sqrt{} - 3n = 4 + \frac{1}{2} \pm (\sqrt{} \frac{25}{4}) \frac{5}{2} =) 7$ vel 2.

3. In Æquatione $x^3 = -15x^2 - 84x + 100$, erit $p = -5$, $q = -3$, $r = 135$; & Binomii $135 + \sqrt{} 18252$ Radix Cubica eſt $3 + \sqrt{} 12$. Igitur Radix $x = -5 + 6 = 1$, & $x = -5 - 3 \pm \sqrt{} - 36 = -8 \pm \sqrt{} - 36$, impoſſibiles.

4. In Æquatione $x^3 = 34x^2 - 310x + 1012$, erit $p = \frac{34}{3}$, $q = \frac{226}{9}$, $r = \frac{5536}{27}$; & Binomii $\frac{5536}{27} + \sqrt{} \frac{707560}{27}$ Radix Cubica eſt $\frac{16}{3} + \sqrt{} \frac{10}{3}$. Igitur Radix $x = \frac{34}{3} + \frac{32}{3} = 22$, & $x = \frac{34}{3} - \frac{16}{3} \pm \sqrt{} - 10 = 6 \pm \sqrt{} - 10$, impoſſibiles.

5. In Æquatione $x^3 = 28x^2 + 61x - 4048$, erit $p = \frac{28}{3}$, $q = \frac{967}{9}$, $r = -\frac{25010}{27}$; & Binomii $-\frac{25010}{27} + \sqrt{} - 382347$. Radix Cubica eſt $\frac{41}{6} + \sqrt{} - \frac{243}{4}$. Igitur $x = \frac{28}{3} + \frac{41}{3} = 23$, & $x = \frac{28}{3} - \frac{41}{6} \pm (\sqrt{} \frac{729}{4}) \frac{27}{2} = 16$ vel -11.

6. In Æquatione $x^3 = -x_2 + 166x - 660$, erit $p = -\frac{1}{3}$, $q = \frac{499}{9}$, $r = -\frac{9658}{27}$; & Binomii $-\frac{9658}{27} + \sqrt{} - \frac{1147205}{27}$ Radix Cubica eſt $-\frac{22}{3} + \sqrt{} - \frac{5}{3}$. Igitur $x = -\frac{1}{3} - \frac{44}{3} = -15$, & $x = -\frac{1}{3} + \frac{22}{3} \pm \sqrt{} 5 = 7 \pm \sqrt{} 5$, irrationales.

7. In

7. In Æquatione $x^3 = 63\,x^2 + 99673\,x + 9951705$, erit $p = 21$, $q = \frac{100996}{3}$, $r = 6031680$; & Binomii $6031680 + \sqrt{-\frac{47887175043136}{27}}$ Radix Cubica eſt $183 + \sqrt{-\frac{529}{3}}$. Igitur $x = 21 + 366 = 387$, & $x = 21 - 183 \pm (\sqrt{529})\,23 = -139$ vel 185.

Nec ſecus in cæteris procedendum: Inveſtigatur autem Theorema ad modum ſequentem. Pono Æquationis cujuſdam Cubicæ Radicem $z = a + b$, & cubicè multiplicando proveniet $z^3 = (a^3 + 3\,a^2\,b + 3\,a\,b^2 + b^3 =)$ $a^3 + 3\,a\,b \times a + b + b^3$. Jam loco ipſius $a + b$ valorem ejus z ſubſtituendo, fiet $z^3 = 3\,a\,b\,z + a^3 + b^3$, quæ eſt Æquatio Cubica ex Radice $z = a + b$ conſtructa, cui terminus ſecundus deeſt. Ut hæc verò ad formam magis commodam magiſq; concinnam revocenter, ſumo Æquationem $z^3 = 3\,q\,z + 2\,r$, quæ poſthac ipſius $z^3 = 3\,a\,b\,z + a^3 + b^3$ vices gerat. Igitur tranſmutatione hujus in illam, fiet primò $3\,q = 3\,a\,b$, ſive $q_3 = a^3\,b^3$; & ſecundò $2\,r = a^3 + b^3$, ſive $2\,r\,a^3 = (a^6 + a^3\,b^3 =) a^6 + q^3$. Et ſoluta hac æquatione quadratica, erit $a^3 = r + \sqrt{r^2 - q^3}$, & hinc $b^3 = (2\,r - a^3 =)\, r - \sqrt{r^2 - q^3}$: Atque igitur tandem $a = \sqrt[3]{r + \sqrt{r^2 - q^3}}$ & $b = \sqrt[3]{r - \sqrt{r^2 - q^3}}$. Et propterea in Æquatione Cubica $z^3 = 3\,q\,z + 2\,r$ erit Radix $z = (a + b =) \sqrt[3]{r^2 + \sqrt{r - q^3}} + \sqrt[3]{r^2 - \sqrt{r - q^3}}$

At verò hæc Radix reverâ triplex eſt, pro triplici valore quem induere poteſt & $\sqrt[3]{r + \sqrt{r^2 - q^3}}$ & $\sqrt[3]{r - \sqrt{r^2 - q^3}}$. Cujuſvis enim quantitatis Radix Cubica triplex erit, & ipſius Unitatis Radix Cubica vel

eſt

eſt 1, vel $-\frac{1}{2}+\frac{1}{2}\sqrt{-3}$, vel $-\frac{1}{2}-\frac{1}{2}\sqrt{-3}$: Atque id adeo, propterea quòd harum alicujus Cubus ſit Unitas. Igitur ſi $1\times\sqrt[3]{r+\sqrt{r^2-q^3}}$ aut $\sqrt[3]{r+\sqrt{r^2-q^3}}$ $(=\sqrt[3]{1\times r+\sqrt{r^2-q^3}}=\sqrt[3]{1}\times\sqrt[3]{r+\sqrt{r^2-q^3}})$ Radicem aliquam [quam ſupra nominavimus $m+\sqrt{n}$, aut $1\times m+\sqrt{n}$,] Cubi $r+\sqrt{r^2-q^3}$ deſignet; ipſæ $\frac{-1+\sqrt{-3}}{2}\times\sqrt[3]{r+\sqrt{r^2-q^3}}$ & $\frac{-1-\sqrt{-3}}{2}\times\sqrt[3]{r+\sqrt{r^2-q^3}}$ [i. e. $\frac{-1+\sqrt{-3}}{2}\times m+\sqrt{n}$ & $\frac{-1-\sqrt{-3}}{2}\times m+\sqrt{n}$] alias duas ejuſdem Cubi Radices deſignabunt. Similiter & $\sqrt[3]{r-\sqrt{r^2-q^3}}$, $\frac{-1+\sqrt{-3}}{2}\times\sqrt[3]{r-\sqrt{r^2-q^3}}$, & $\frac{-1-\sqrt{-3}}{2}\times\sqrt[3]{r-\sqrt{r^2-q^3}}$, [i. e. $m-\sqrt{n}$, $\frac{-1+\sqrt{-3}}{2}\times m-\sqrt{n}$, $\frac{-1-\sqrt{-3}}{2}\times m-\sqrt{n}$,] tres Cubicæ Radices erunt Apotomes $r-\sqrt{r^2-q^3}$. Atque has Radices debitè connectendo, fiet $z=\sqrt[3]{r+\sqrt{r^2-q^3}}+\sqrt[3]{r-\sqrt{r^2-q^3}}$, [i. e. $z=m+\sqrt{n}+m-\sqrt{n}=2m$,] $z=\frac{-1+\sqrt{-3}}{2}\times\sqrt[3]{r+\sqrt{r^2-q^3}}+\frac{-1-\sqrt{-3}}{2}\times\sqrt[3]{r-\sqrt{r^2-q^3}}$, [i. e. $z=\frac{-1+\sqrt{-3}}{2}\times m+\sqrt{n}+\frac{-1-\sqrt{-3}}{2}\times m-\sqrt{n}=-m+\sqrt{-3n}$,] & $z=\frac{-1-\sqrt{-3}}{2}\times\sqrt[3]{r+\sqrt{r^2-q^3}}+\frac{-1+\sqrt{-3}}{2}\times\sqrt[3]{r-\sqrt{r^2-q^3}}$

[i. e. $z = \frac{-1-\sqrt{-3}}{2} \times \sqrt[3]{m+\sqrt{n}} + \frac{-1+\sqrt{-3}}{2} \times \sqrt[3]{m-\sqrt{n}} = -m-\sqrt{-3n}$,] quæ tres erunt Radices Æquationis Cubicæ $z^3 = 3qz + 2r$. Debite autem connectuntur Radices iſtæ ad modum præcedentem, quippe quæ ſic connexæ, & more vulgari in ſe invicem continue ductæ, Æquationem $z^3 = 3qz + 2r$ reſtituunt. Denique fac $z = x - p$, & Æquatio fiet $x^3 - 3px^2 + 3p^2x - p^3 = 3qx - 3pq + 2r$, quæ univerſalis eſt, & cujus Radices evadunt ut ſupra fuerunt exhibitæ.

Hic obiter notatu dignum eſt, quod Æquationis Cubicæ cujuſcunque Radices omnes ſint poſſibiles & reales, quoties Binomii membrum irrationale $\sqrt{r^2 - q^3}$ impoſſibilitatem in ſe complectitur; hoc eſt, quoties q eſt quantitas affirmativa, & ſimul cubus ejus major eſt quadrato ex latere r. At ſi membrum iſtud $\sqrt{r^2 - q^3}$ ſit poſſibile, hoc eſt ſi q ſit quantitas negativa, aut etiam ſi affirmativæ cubus ſit minor quadrato ex latere r, tunc unicam tantùm agnoſcit Æquatio Radicem poſſibilem & realem, reliquæque duæ erunt impoſſibiles.

In hoc Theoremate ſi fiat $p = 0$, hoc eſt, ſi deſit Æquationis terminus ſecundus, tunc deventum erit ad caſum Regularum quæ dicuntur *Cardani*, cujus ſolutio continetur in præcedentibus.

§. 2. Æquationis Biquadraticæ Univerſalis

$$x^4 = 4px^3 + 2qx^2 + 8rx + 4s,$$
$$-4p^2 - 4pq - q^2$$

Radices quatuor ſunt $x = p - a \pm \sqrt{p^2 + q - a^2 - \frac{2r}{a}}$,

& $x = p + a \pm \sqrt{p^2 + q - a^2 + \frac{2r}{a}}$, Ubi a^2 eſt Radix Æquationis Cubicæ $a^6 = p^2a^4 - 2pra^2 + r^2$.
$+q \quad -s$

Jam data Æquatione quavis Biquadratica, inter ejus hujuſque Æquationis Univerſalis terminos ſingulos inſtitu-

enda

enda eſt comparatio, quo pacto citiſſime invenientur ipſæ p, q, r, s; & hiſce cognitis, non latebit valor ipſius a, ex Theoremate ſuperiori inveniendus, & tum demum innoteſcent Æquationis datæ Radices omnes.

Huic Solutioni illuſtrandæ Exemplum unum aut alterum ſufficiat.

1. Æquationis Biquadraticæ $x^4 = 8x^3 + 83x^2 - 162x - 936$ ſint Radices extrahendæ. Erit primò juxta præſcriptum $4p = 8$, ſive $p = 2$. Secundò $2q - (4p^2) 16 = 83$, ſive $q = \frac{99}{2}$. Tertiò $8r - (4pq) 396 = -162$, ſive $r = \frac{117}{4}$. Quartò $4s - (q^2) \frac{9801}{4} = -936$, ſive $s = \frac{6057}{16}$. Hinc $p^2 + q = \frac{107}{2}$, $2pr + s = \frac{7929}{16}$, $r^2 = \frac{1368_9}{19}$, & proinde $a^6 = \frac{107}{2}a^4 - \frac{7929}{16}a^2 + \frac{1368_9}{16}$. Jam ut Æquatio hæc aliquatenus Cubica in Radices ejus reſolvatur, ad Theorema præcedens recurrendum eſt, in quo erit $p = \frac{107}{2}$, $q = \frac{22009}{144}$, $r = \frac{2903923}{1728}$ & $r^2 - q^3 = -\frac{11940075}{16}$. Atqui Binomii $\frac{2903923}{1728} + \sqrt{-\frac{11940075}{16}}$ Radix Cubica eſt $-\frac{53}{12} + \sqrt{-\frac{400}{3}}$ & propterea $a^2 = \frac{107}{6} - \frac{53}{6} = 9$, & etiam $a^2 = \frac{107}{6} + \frac{53}{12} \pm (\sqrt{400})\ 20 = \frac{169}{4}$ vel $\frac{9}{4}$: Vel quod perinde eſt, Æquationis præmiſſæ reverà Cubo-Cubicæ ſex Radices ſunt $a = \pm 3$, $a = \pm \frac{13}{2}$, & $a = \pm \frac{3}{2}$, quarum quævis indiſcriminatim propoſito

ſito noſtro faciet ſatis. Puta ſi in præſenti caſu fiat $a = 3$, erit juxta Theorema $x = (p - a \pm \sqrt{p^2 + q - a^2 - \frac{2r}{a}} = 2 - 3 \pm \sqrt{4 + \frac{99}{2} - 9 - \frac{39}{2}}$ $= -1 \pm (\sqrt{25})\,5 =) 4$ vel -6, & $x = (p + a \pm \sqrt{p^2 + q - a^2 + \frac{2r}{a}} = 2 + 3 \pm \sqrt{4 + \frac{99}{2} - 9 + \frac{39}{2}}$ $= 5 \pm (\sqrt{64})\,8 =) 13$ vel -3, quæ ſunt Æquationis datæ Radices quatuor.

2. In Æquatione $x^4 = 20x^3 + 252x^2 - 6592x + 21312$, erit $p = 5$, $q = 176$, $r = -384$, & $s = 13072$. Hinc $p^2 + q = 201$, $2pr + s = 9232$, & $r^2 = 147456$; & inde $a^6 = 201\,a^4 - 9232\,a^2 + 147456$. Jam in Theoremate pro Cubicis erit $p = 67$, $q = \frac{4235}{3}$, & $r = 65219$; eritque Binomii $65219 + \sqrt{\frac{38889307072}{27}}$ Radix Cubica $\frac{77}{2} + \sqrt{\frac{847}{12}}$. Igitur $a^2 = 67 + 77 = 144$, ſive $a = 12$; & proinde $x = 5 - 12 + \sqrt{25 + 176 - 144 + 64} = -7 \pm (\sqrt{121})\,11 = 4$ vel -18, & $x = 5 + 12 \pm \sqrt{25 + 176 - 144 - 64} = 17 \pm \sqrt{-7}$, impoſſibiles.

Hujus autem Theorematis Inventio eſt hujuſmodi, Ex duarum Æquationum Quadraticarum $z^2 + 2az - b = 0$, & $z^2 - 2az - c = 0$ in ſe invicem multiplicatione, Æquationem conficio Biquadraticam $z^4 = 4a^2 + b + c \times z^2 + 2ac - 2ab \times z - bc$, cui terminus ſecundus deeſt, quamque hujuſce Æquationi $z^4 = ez^2 + fz + g$ ſtatuo æquipolere. Unde primo $4a^2 + b + c = e$ ſive $b = e - 4a^2 - c$. Secundò $2ac - 2ab = f$, hoc eſt, $2ac - 2ae + 8a^3 + 2ac = f$, ſive $c = \frac{f}{4a} + \frac{e}{2} - 2a^2$,

&

& inde $b = (e - 4a^2 - c =) - \frac{f}{4a} + \frac{e}{2} - 2a^2$. Tertiò $-bc = g$, ſive $-\frac{f^2}{16a^2} + \frac{e^2}{4} - 2ea^2 + 4a^4 = -g$, hoc eſt, $a^6 = \frac{1}{2}ea^4 - \frac{1}{4}ga^2 - \frac{1}{16}ea^2 + \frac{f^2}{64}$, quæ Æquatio quaſi Cubica eſt, ex Radice a^2 & notis vel aſſumptis e, f, g conſtata. Ea verò Radix per Theorema ſuperius exhiberi poteſt, & eodem Calculo innoteſcent ipſæ b & c. At Æquationum $z^2 + 2az - b = 0$ & $z^2 - 2az - c = 0$ Radices ſunt $z = -a \pm \sqrt{a^2 + b}$ & $z = a \pm \sqrt{a^2 + c}$, ſive $z = -a \pm \sqrt{\frac{1}{2}e - a^2 - \frac{f}{4a}}$, & $z = a \pm \sqrt{\frac{1}{2}e - a^2 + \frac{f}{4a}}$, quæ proinde erunt Radices Æquationis $z^4 = ez^2 + fz + g$; cognita videlicet a vel a^2 ex Æquatione $a^6 = \frac{1}{2}ea^4 - \frac{1}{4}ga^2 - \frac{1}{16}ea^2 + \frac{f^2}{64}$. Jam ut Æquatio iſta evadat univerſalis, & omnibus ſuis terminis inſtructa, fac. $z = x - p$, eritque $x^4 - 4px^3 + 6p^2x^2 - 4p^3x + p^4 = ex^2 - 2pex + p^2e + fx - fp + g$, item & $x = p - a \pm \sqrt{\frac{1}{2}e - a^2 - \frac{f}{4a}}$, & $x = p + a \pm \sqrt{\frac{1}{2}e - a^2 + \frac{f}{4a}}$. Tandem concinnitatis & compendii gratiâ, fac $e = 2q + 2p^2$ & $f = 8r$; tum $x^4 - 4px^3 + 4p^2x^2 = 2qx^2 - 4pqx + 2p^2q + p^4 + 8rx - 8pr + g$, $x = p - a \pm \sqrt{p^2 + q - a^2 - \frac{2r}{a}}$, $x = p + a \pm \sqrt{p^2 + q - a^2 + \frac{2r}{a}}$, & $a^6 = p^2 + q \times a^4 - \frac{1}{4}g + \frac{1}{4}p^4 + \frac{1}{2}p^2q - \frac{1}{4}q^2 \times a^2 + r^2$. Denique fac $g = 4s - q^2 + 8pr - p^4 - 2p^2q$, & fiunt Æquationes præcedentes $x^4 = 4px^3 + 2qx^2 + 8rx + 4s$ & $a^6 = p^2a^4 - 2pra^2 + r^2$.
$-4p^2 \quad -4pq - q^2 \quad +q \quad -s$
Scilicet omnia evadunt ut ſupra ſunt poſita.

§ 8. Hactenus de Æquationum Cubicarum & Biquadraticarum Reſolutione Analytica. Quoniam autem earundem *Effectio Geometrica* per Parabolam vulgò tradi ſolet, & nonnullis in pretio eſt, ipſam συνοπτικῶς, & quidem univerſalius, non pigebit hic exhibere.

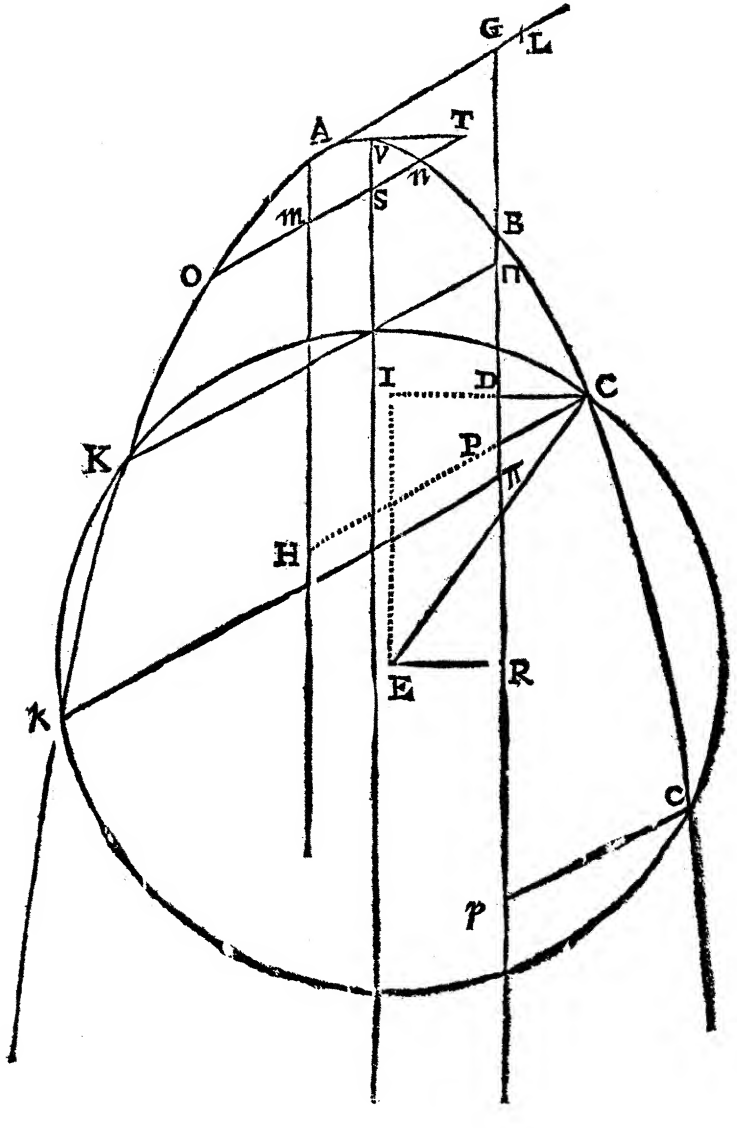
Data Æquatione quavis vel Cubica vel Biquadratica, inſtituenda eſt comparatio inter terminos ejus, terminoſque reſpondentes hujus Æquationis

$$x^4 = \frac{2p}{q}x^3 + \frac{4pr}{q}x^2 + \frac{2p^2}{q}x + p^2,$$

quo pacto facile ſatis

$$\begin{array}{llll} -4r & -4r^2 & -\frac{2ps}{q} & -q^2 \\ & +2s & +4rs & -s^2 \\ & -1 & -2q & +t^2 \end{array}$$

eruentur ipſæ p, q, r, s, t; earum interim unâ aliquâ utcunque pro lubitu aſſumptâ. Tum in Parabola quavis data AVB, cujus Vertex principalis V, Axis VS, & Axi



per-

perpendicularis VT; capiatur VS = p verſus interiora Parabolæ, & in angulo SVT inſcribatur ST = q, quæ producta Parabolam ſecet in punctis binis N & O. Biſecetur ON in M, & per M agatur MA Axi parallela & Parabolæ occurrens in A. Ipſi ON parallela ducatur AL, ut ſit AL Latus rectum Parabolæ ad Diametrum AM, ſitque hæc eadem Unitas. In AL (utrinque ſi opus eſt producta) capiatur AG = r, & à puncto G ducatur GR Axi parallela, quæ Parabolam ſecet in B, à quo capiatur BR = s. A noviſſime invento puncto R ducatur RE ipſi VT parallela & æqualis, quæ ſiniſtram verſus jaceat reſpectu ipſius R ſi q ſit quantitas affirmativa, at verſus dextram ſi q ſit negativa. Atque idem de ipſis AG & BR intelligatur, quæ ad contrarias itidem partes duci debent, ſi modò valores ipſarum r & s prodeant negativi. Denique Centro E & Radio EC = t deſcribatur Circulus CKκc, qui Parabolam in totidem ſecabit punctis, quot ſunt Æquationis datæ Radices reales. Etenim à punctis iſtis C, K, &c. ducantur CP, κπ, &c. ipſi ST parallelæ, & ad rectam GR (ſi opus eſt productam) terminatæ, eritque harum quævis x, ſeu Æquationis datæ Radix quæſita; eæ ſcilicet ad dextram jacentes erunt Radices affirmativæ, quæ verò ad ſiniſtram ſunt poſitæ erunt Radices negativæ. Punctum contactûs, ſiquod fuerit, hic ſumitur pro interſectionis punctis duobus ad invicem viciniſſimis.

Inter Æquationes Cubicas & Biquadraticas ita conſtructas hoc tantùm intercedit diſcriminis, quòd in prioribus, ob terminum ultimum in præcedente Æquatione deficientem, ſemper ſit $p^2 - q^2 - s^2 + t^2 = 0$, ſive $t = \sqrt{s^2 + q^2 - p^2}$. Igitur Centro E & Radio EB ($= \sqrt{BRq + (ERq) STq - VSq}$) deſcripto Circulo CKκc, Radicum una CP in priori conſtructione in nihilum abit.

Hæc autem demonſtrantur ad modum ſequentem. Manentibus jam conſtructis, & productâ CP ſi opus eſt, donec ſecat AM in H, erit CH Ordinata Parabolæ ad Diametrum

metrum AH, & proinde $CHq = AL \times AH = AH$, ob $AL = 1$. At $CH = CP + AG$, & $AH = GB + BP$, & propterea $CPq + 2AG \times CP + AGq = GB + BP$; ſed ob naturam Parabolæ erit $AGq = GB$, unde $CPq + 2AG \times CP = BP$. Jam à puncto C ad ipſam BP demittatur norma sCD, quæ occurrat etiam ipſi EI, ad BP actæ parallelæ, in puncto I. Propter ſimilia Triangula CDP & TVS, erit $DP = \frac{VS \times CP}{ST}$ & $CD = \frac{VT \times CP}{ST}$, & proinde $CPq + 2AG \times CP = BP = DP + BD = \frac{VS \times CP}{ST} + BR - IE$, ſive $CPq + 2AG \times CP - \frac{VS}{ST} CP - BR = -IE$. Aſt $IEq = CEq - CIq = CEq - CDq - VTq - 2CD \times VT = CEq - \frac{VTq \times CPq}{STq} - VTq - \frac{2VTq \times CP}{ST} = (\text{ob } VTq = STq - SVq)\ CEq - CPq + \frac{SVq}{STq} CPq - STq + SVq - 2ST \times CP + \frac{2SVq}{ST} CP$, quæ igitur æqualis erit Quadrato ex Latere $CPq + 2AG \times CP - \frac{VS}{ST} CP - BR$. Atque hæc Æquatio ad terminos p, q, r, s, t revocata ipſiſſima ſit Æquatio propoſita.

Hinc liquet, quòd eadem quævis Æquatio Biquadratica innumeras per Parabolam conſtructiones ſortiri poſſit, pro indefinito valore quantitatis iſtius, quam ad arbitrium aſſumi poſſe jam diximus. Sed caſus eſt ſimpliciſſimus faciendo $VS = p = o$, & migrat conſtructio, ſi rem ipſam ſpectes, in vulgarem iſtam, in qua Radicum repræſentatrices rectæ CP, &c. ſunt ad Axem perpendiculares. Æquatio autem fit $x^4 = -4rx^3 - 4r^2x^2 + 4rsx - q^2$, quæ facile

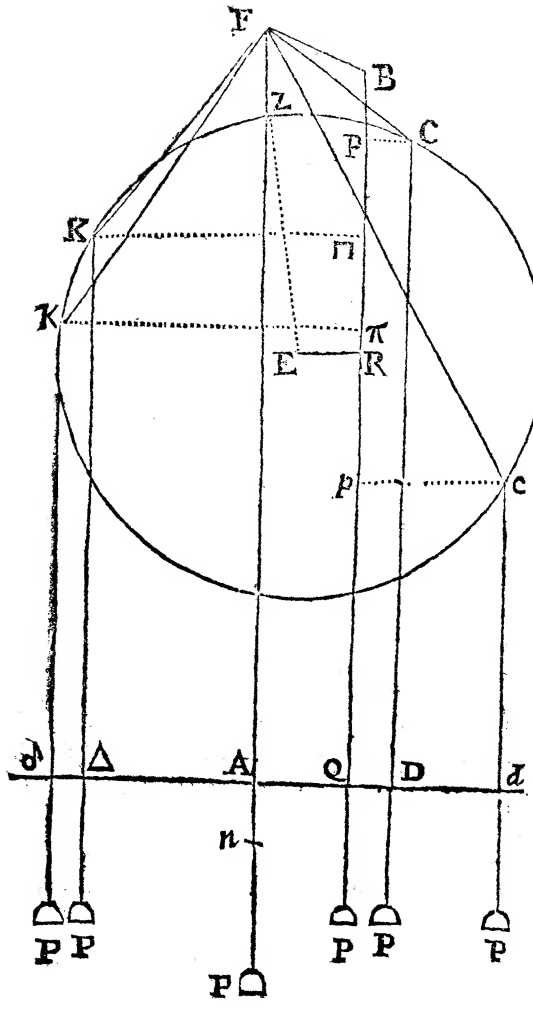
$+2s \quad = 2q \quad - s^2$

$-1 \quad\quad + t^2$

conſtruitur ut ſupra.

§ 4. Sed ne Parabolæ deſcriptio Organica difficilis nimium videatur, in promptu eſt Artificium quoddam Mechanicum, ope Fili penduli pondere inſtructi peractum, cujus auxilio quam exactiſſime & facillime Æquatio noviſſima conſtrui poteſt, & proinde Æquationum quarumcunque Cubicarum & Biquadraticarum Radices inveniri; idque ſine ullo linearum ductu niſi Rectarum & Circuli. Conſtructio autem, quam appellare libet *Mechanicam*, eſt ad hunc modum.

Contra Parietem erectum, vel planum aliud quodvis Horizonti perpendiculare, ad punctum aliquod F ſuſpendatur filum tenuiſſimum & flexile FP; pondere quovis P ad extremitatem P appenſo. In hoc filo notetur punctum aliquod N, à puncto ſuſpenſionis F ſatis remotum; vel filo parvulus, ſi id mavis, innectatur Nodus N. Et ſumpta utcunque NO pro Unitate, ad punctum medium A ducatur (in plano prædicto) recta AQ Horizonti parallela, & utrinque quantum ſatis producta. Hiſce generaliter paratis, pro particulari jam applicatione fac AQ = r; ipſis q, r, s, t, ut ſæpius inculcatum, vel Arithmeticè vel Geometricè, pro datæ cujuſvis Æquationis ex-



exigentia, in Æquatione novillima priùs determinatis. Tunc Acu vel Stylo tenuiſſimo, aut etiam cuſpide Circini admodum gracili, flectatur filum à loco ſuo ad punctum quoddam B, ita ut punctum N cadat in noviſſime invento puncto Q. In BQ ab iſto B capiatur BR = s, & in R ad ipſam BR perpendicularis erigatur ER = q. Verùm enimverò iſtæ AQ, BR, RE ad contrarias partes ab earum initiis cadere debent, ſi fortè valores ipſarum r, s, q prodeant negativi. Denique in puncto invento E figatur Circini crus unum, &, ad diſtantiam EZ = t extentum, agatur crus alterum in orbem, ſecumque circumducat filum FZP. Hac fili circulatione pondus P nunc aſcendet nunc deſcendet motu reciproco, ut & Nodus N nunc ſupra rectam AQ extabit, nunc verò infra eandem deprimetur. Quoties autem reperietur Nodus ille N in ipſa AQ, puta in punctis D, d, Δ, δ, ab ſcindet is rectas DQ, dQ, ΔQ, δQ.

quæ erunt Æquationis datæ Radices omnes reales; hæ nempe ad dextram erunt Radices affirmativæ, illæ verò ad ſiniſtram Radices negativæ. Demonſtratio eſt manifeſta ex præcedentibus, habita tantùm ratione Parabolæ per puncta B, C, c, κ, κ tranſeuntis. Nam poſito F foco Parabolæ, (cujus diſtantia à Vertice aſt $\frac{1}{4}$ ON,) notum eſt quod lineæ omnes ut FB + BQ, FC + CD, &c, eandem ubique conficiant ſummam.

Atque ex principiis hic poſitis proclive erit Inſtrumentum haud inconcinnum & quantumvis accuratum fabricari, cujus beneficio hujuſmodi Æquationum quarumcunque Radices nullo fere negotio inveniri poſſint, & præ oculis exhiberi. Hoc autem quilibet, ſi id Curæ ſit, variis modis pro ingenio ſuo efficere poteſt, & de his jam ſatis.

III. *Æquationum quarundam Poteſtatis tertiæ, quintæ, ſeptimæ, nonæ, & ſuperiorum, ad infinitum uſque pergendo, in terminis finitis, ad inſtar Regularum pro Cubicis quæ vocantur* Cardani, *Reſolutio Analytica.*

Per Ab. De Moivre, R. S. S.

SIt n Numerus quicunque, y quantitas incognita, ſive Æquationis Radix quæſita, ſitque a quantitas quævis omnino cognita, ſive ut vocant Homogeneum Comparationis: Atque horum inter ſe relatio exprimatur per Æquationem

$$ny + \frac{nn-1}{2\times 3}ny^3 + \frac{nn-1}{2\times 3}\times\frac{nn-9}{4\times 5}ny^5 + \frac{nn-1}{2\times 3}\times\frac{nn-9}{4\times 5}\times\frac{nn-25}{6\times 7}ny^7, \text{ &c.} = a$$

Ex